

Latihan Soal Bab 5

1. Dapatkan persamaan garis singgung di $t = \frac{\pi}{6}$ pada kurva $x = 2 \sin t$ dan $y = t + \cos t$

Jawab :

► Menentukan titik singgung

$$\begin{aligned} x &= 2 \sin\left(\frac{\pi}{6}\right) & y &= \frac{\pi}{6} + \cos\left(\frac{\pi}{6}\right) \\ &= 2 \cdot \sin 30^\circ & &= \frac{\pi}{6} + \frac{\sqrt{3}}{2} \\ &= 2 \cdot \frac{1}{2} & & \\ &= 1 & & \end{aligned}$$

► Menentukan Gradien Garis Singgung

$$d_m = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1 - \sin t}{2 \cos t}$$

$$\begin{aligned} \text{Subst di } t = \frac{\pi}{6} \rightarrow m &= \frac{1 - \sin 30^\circ}{2 \cdot \cos 30^\circ} \\ &= \frac{1 - \frac{1}{2}}{2 \cdot \frac{\sqrt{3}}{2}} \\ &= \frac{\frac{1}{2}}{\sqrt{3}} \\ &= \frac{1}{2\sqrt{3}} \end{aligned}$$

► Menentukan persamaan garis Singgung

$$y - y_1 = m(x - x_1)$$

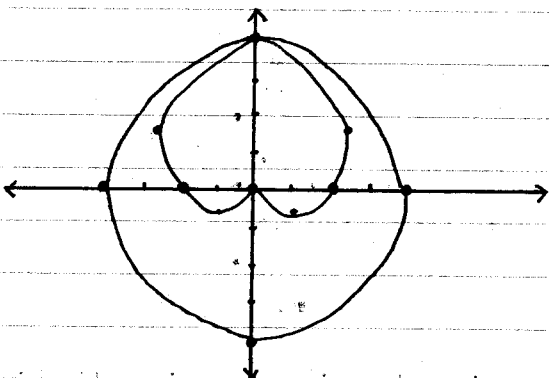
$$y - \left[\frac{\pi}{6} + \frac{\sqrt{3}}{2}\right] = \frac{1}{2\sqrt{3}}(x - 1)$$

$$y = \frac{1}{6}\sqrt{3}(x - 1) + \left[\frac{\pi}{6} + \frac{\sqrt{3}}{2}\right]$$

- 2). Dapatkan luas daerah di luar kardioida $r = 2(1 + \sin \theta)$ dan di dalam lingkaran $r = 4$

Jawab :

θ	0°	30°	90°	150°	180°	210°	270°	330°	360°
r	2	3	4	3	2	1	0	1	2



$$\begin{aligned} L &= \int_0^{2\pi} \frac{1}{2} \left[4^2 - [2 + 2\sin \theta]^2 \right] d\theta \\ &= \int_0^{2\pi} \frac{1}{2} \left[16 - (4 + 8\sin \theta + 4\sin^2 \theta) \right] d\theta \end{aligned}$$

$$= \frac{1}{2} \int_0^{2\pi} 12 - 8\sin \theta - 4\sin^2 \theta d\theta$$

$$= \int_0^{2\pi} 6 - 4\sin \theta - 2 \left[\frac{1}{2} (1 - \cos 2\theta) \right] d\theta$$

$$= \int_0^{2\pi} 6 - 4\sin \theta - 1 + \cos 2\theta d\theta$$

$$= \int_0^{2\pi} 5 - 4\sin \theta + \cos 2\theta d\theta$$

$$= 5\theta + 4\cos \theta + \frac{1}{2} \sin 2\theta \Big|_0^{2\pi}$$

$$= 10\pi + 4 + \frac{1}{2} \sin(4\pi) - \left[0 + 4(1) + \frac{1}{2} \sin(0^\circ) \right]$$

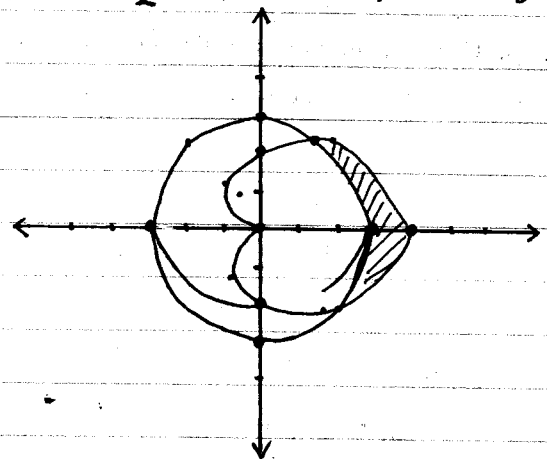
$$= 10\pi + 4 + 0 - (4)$$

$$= 10\pi \text{ Satuan luas}$$

- 3). Tentukan luas daerah di luar lingkaran $r = 3$, dan di dalam kardioida $r = 2 + 2\cos \theta$

Jawab :

θ	0°	60°	90°	150°	180°	240°	270°	300°	360°
r	4	3	2	1	0	1	2	3	4



Titik Potong :

$$r_1 = r_2$$

$$3 = 2 + 2\cos \theta$$

$$2\cos \theta = 1$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = 60^\circ, 300^\circ$$

$$\begin{aligned}
 L &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} [(2+2\cos\theta)^2 - 3^2] d\theta \\
 &= 2 \int_0^{\pi/2} \frac{1}{2} [4 + 8\cos\theta + 4\cos^2\theta - 9] d\theta \\
 &= \int_0^{\pi/2} [4\cos^2\theta + 8\cos\theta - 5] d\theta \\
 &= \int_0^{\pi/2} \left[4 \cdot \frac{1}{2}(1+\cos 2\theta) + 8\cos\theta - 5 \right] d\theta \\
 &= \int_0^{\pi/2} [2 + 2\cos 2\theta + 8\cos\theta - 5] d\theta \\
 &= \int_0^{\pi/2} [-3 + 8\cos\theta + 2\cos 2\theta] d\theta \\
 &= -3\theta + 8\sin\theta + 2 \cdot \frac{1}{2} \sin 2\theta \Big|_0^{\pi/2} \\
 &= -3 \cdot \frac{\pi}{2} + 8 \sin \frac{\pi}{2} + \sin \frac{2\pi}{2} - 0 \\
 &= -\frac{3\pi}{2} + 8 \cdot \frac{1}{2} + \frac{1}{2} \cdot 0 \\
 &= \frac{9}{2} - \frac{3\pi}{2}
 \end{aligned}$$

$$L_{\text{luas}} = \left(\frac{9}{2} - \frac{3\pi}{2} \right) \text{ satuan luas}$$

4). Dapatkan panjang busur dari kurva $x=t^2$
dan $y=\frac{1}{3}t^3$ sepanjang interval $0 \leq t \leq 1$

Jawab :

$$\frac{dx}{dt} = 2t \quad \frac{dy}{dt} = t^2$$

$$\begin{aligned}
 S &= \int_0^1 \sqrt{(2t)^2 + (t^2)^2} dt \\
 &= \int_0^1 \sqrt{4t^2 + t^4} dt \\
 &= \int_0^1 \sqrt{t^2(4+t^2)} dt \\
 &= \int_0^1 t \sqrt{4+t^2} dt
 \end{aligned}$$

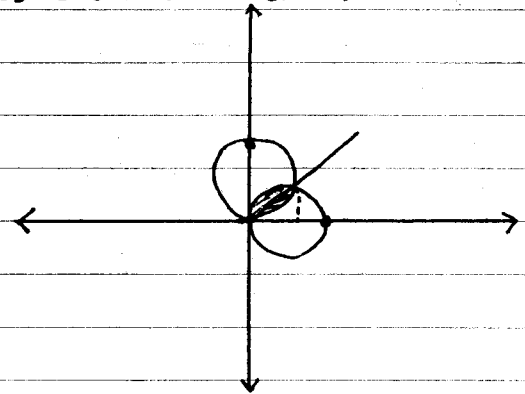
Misal : $u = 4 + t^2$

$$du = 2t dt \rightarrow dt = \frac{du}{2t}$$

$$BA : u = 4+1 = 5 \quad BB : u = 4+0 = 4$$

$$\begin{aligned}
 S &= \int_4^5 \sqrt{u} \frac{du}{2} \\
 &= \frac{2}{3} u^{3/2} \Big|_4^5 \\
 &= \frac{2}{3} [5\sqrt{5} - 8]
 \end{aligned}$$

5). Luas daerah irisan dari $r = 2\sin\theta$ dan $r = 2\cos\theta$



Titik Potong :

$$r_1 = r_2$$

$$2\sin\theta = 2\cos\theta$$

$$\tan\theta = 1$$

$$\theta = \frac{\pi}{4}$$

$$L_{\text{tot}} = L_1 + L_2 \rightarrow 2L_2 \text{ (karena Simetris)}$$

$$\begin{aligned}
 L_1 &= \int_0^{\pi/4} \frac{1}{2} (2\sin\theta)^2 d\theta \\
 &= \frac{1}{2} \int_0^{\pi/4} 4\sin^2\theta d\theta \\
 &= 2 \int_0^{\pi/4} \frac{1}{2} (1 - \cos 2\theta) d\theta \\
 &= \int_0^{\pi/4} 1 - \cos 2\theta d\theta \\
 &= \theta - \frac{1}{2} \sin 2\theta \Big|_0^{\pi/4} \\
 &= \frac{\pi}{4} - \frac{1}{2}
 \end{aligned}$$

$$L_{\text{tot}} = 2 \times L_1 = 2 \cdot \left[\frac{\pi}{4} - \frac{1}{2} \right]$$

$$= \frac{\pi}{2} - 1$$

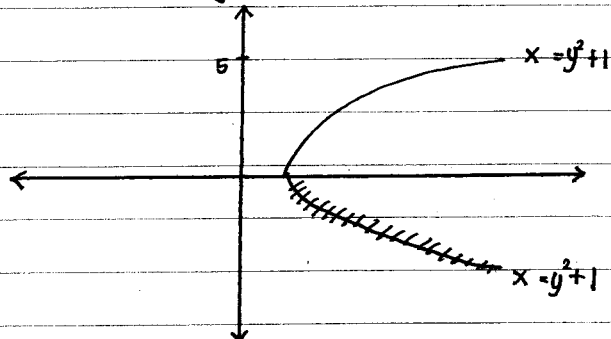
6). Persamaan parametrik $x = t^2 + 1$, $y = t$,

untuk $0 \leq t \leq 5$

a. Eliminasi parameter t

karena $y = t$, maka:

$$x = y^2 + 1$$



hanya ambil $0 \leq t \leq 5$ sesuai interval

b. Persamaan Garis Singgung $\rightarrow t = \frac{1}{2}$

$$m = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1}{2t}$$

$$\text{Saat } t = \frac{1}{2} \rightarrow m = \frac{1}{1} = 1$$

$$t = \frac{1}{2} \rightarrow x = \frac{1}{4} + 1 \quad \left| \quad y = \frac{1}{2} \right. \\ = \frac{5}{4}$$

► Persamaan Garis Singgung :

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{2} = (x - \frac{5}{4})$$

$$y = x - \frac{5}{4} + \frac{1}{2}$$

$$y = x - \frac{3}{4}$$

$$4y = 4x - 3$$

$$4y - 4x + 3 = 0$$

► Titik Potong

$$r_1 = r_2$$

$$2 - 2\cos\theta = 2 + 2\cos\theta$$

$$0 = 4\cos\theta$$

$$\cos\theta = 0$$

$$\theta = 90^\circ, 270^\circ$$

Menentukan Rumus Luas :

$$L_{\text{tot}} = 4 \times L_1$$

$$L_1 = \int_0^{\pi/2} \frac{1}{2} (2 - 2\cos\theta)^2 d\theta$$

$$= \frac{1}{2} \int_0^{\pi/2} 4 - 8\cos\theta + 4\cos^2\theta d\theta$$

$$= \frac{1}{2} \left[\int_0^{\pi/2} 4 - 8\cos\theta + 4 \left[\frac{1}{2} (1 + \cos 2\theta) \right] d\theta \right]$$

$$= \frac{1}{2} \left[\int_0^{\pi/2} 4 + 2 - 8\cos\theta + 2\cos 2\theta d\theta \right]$$

$$= \int_0^{\pi/2} 3 - 4\cos\theta + \cos 2\theta d\theta$$

$$= 3\theta - 4\sin\theta + \frac{1}{2}\sin 2\theta \Big|_0^{\pi/2}$$

$$= \frac{3\pi}{2} - 4 + 0 - (0 - 0 + 0)$$

$$= \frac{3\pi}{2} - 4$$

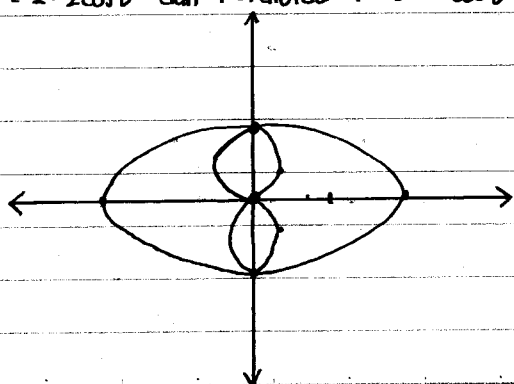
$$L_{\text{total}} = 4 \times L_1$$

$$= 4 \left[\frac{3\pi}{2} - 4 \right]$$

$$= 6\pi - 16 //$$

7). Dapatkan luas daerah dari irisan kardioida

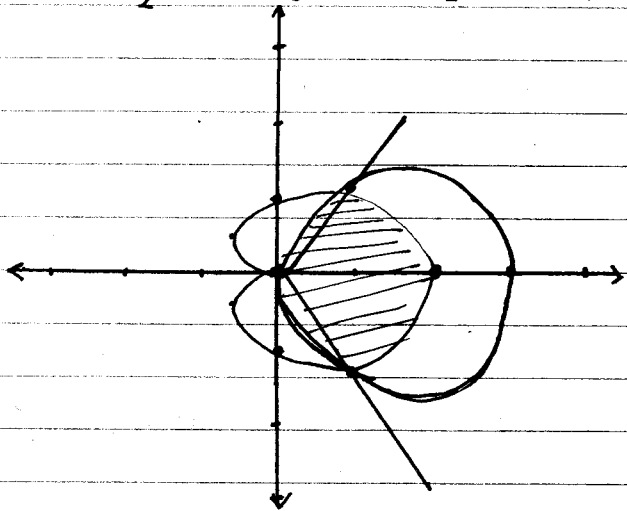
$r = 2 - 2\cos\theta$ dan kardioida $r = 2 + 2\cos\theta$



8). Dapatkan luas daerah di dalam lingkaran $r = 3\cos\theta$ dan di dalam Kardoida $r = 1 + \cos\theta$

① Menentukan Gambar Luasan

θ	0°	60°	90°	120°	180°	240°	270°	300°	360°
r_1	3	$\frac{3}{2}$	0	$-\frac{3}{2}$	-3	$-\frac{3}{2}$	0	$\frac{3}{2}$	3
r_2	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2



② Menentukan Titik Potong

$$\begin{aligned} r_1 &= r_2 \\ 3\cos\theta &= 1 + \cos\theta \\ 2\cos\theta &= \frac{1}{2} \\ \cos\theta &= \frac{1}{2} \rightarrow \theta = 60^\circ, 300^\circ \end{aligned}$$

③ Merumuskan Luas :

Luasnya cukup hitung daerah atas, karena bagian luasan atas sb. x simetri dengan luasan bawah sumbu x.

$$\begin{aligned} L_{\text{tot}} &= 2 \times L_{\text{atas}} \\ L_{\text{atas}} &= \int_0^{\pi/6} \frac{1}{2} (1 + \cos\theta)^2 d\theta + \int_{\pi/6}^{\pi/2} \frac{1}{2} (3\cos\theta)^2 d\theta \\ &= \frac{1}{2} \int_0^{\pi/6} 1 + 2\cos\theta + \cos^2\theta d\theta + \frac{1}{2} \int_{\pi/6}^{\pi/2} 9\cos^2\theta d\theta \\ &= \frac{1}{2} \int_0^{\pi/6} 1 + 2\cos\theta + \frac{1}{2}(1 + \cos 2\theta) d\theta + \frac{9}{2} \int_{\pi/6}^{\pi/2} \frac{1 + \cos 2\theta}{2} d\theta \\ &= \frac{1}{2} \int_0^{\pi/6} 1 + \frac{1}{2} + 2\cos\theta + \frac{1}{2}\cos 2\theta d\theta + \frac{9}{4} \int_{\pi/6}^{\pi/2} 1 + \cos 2\theta d\theta \\ &\quad \underbrace{\hspace{10em}}_{L_1} \quad \underbrace{\hspace{10em}}_{L_2} \end{aligned}$$

$$\begin{aligned} * L_1 &= \frac{1}{2} \int_0^{\pi/6} \left[\frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos 2\theta \right] d\theta \\ &= \frac{1}{2} \left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta \right]_0^{\pi/6} \\ &= \frac{1}{2} \left[\frac{\pi}{2} + 2\left(\frac{1}{2}\right) + \frac{1}{4}\sqrt{3} - 0 \right] \\ &= \frac{1}{2} \left[\frac{\pi}{2} + \sqrt{3} + \frac{1}{4}\sqrt{3} \right] \\ &= \frac{1}{2} \left[\frac{\pi}{2} + \frac{9}{8}\sqrt{3} \right] \end{aligned}$$

$$\begin{aligned} * L_2 &= \frac{9}{4} \int_{\pi/6}^{\pi/2} 1 + \cos 2\theta d\theta \\ &= \frac{9}{4} \left[\theta + \frac{1}{2}\sin 2\theta \right]_{\pi/6}^{\pi/2} \\ &= \frac{9}{4} \left[\frac{\pi}{2} + 0 - \left(\frac{\pi}{6} + \frac{1}{2} \cdot \frac{1}{2}\sqrt{3} \right) \right] \\ &= \frac{9}{4} \left[\frac{\pi}{6} - \frac{1}{4}\sqrt{3} \right] \end{aligned}$$

$$= \frac{3\pi}{8} - \frac{9}{16}\sqrt{3}$$

$$L_{\text{total}} = 2 \times (L_1 + L_2)$$

$$= 2 \times \left[\frac{\pi}{4} + \frac{9}{16}\sqrt{3} + \frac{3\pi}{8} - \frac{9}{16}\sqrt{3} \right]$$

$$= \frac{5\pi}{4}$$

$$= 2 \times \left[\frac{\pi}{4} + \frac{9}{16}\sqrt{3} + \frac{3\pi}{8} - \frac{9}{16}\sqrt{3} \right]$$

$$= 2 \cdot \left[\frac{2\pi}{8} + \frac{3\pi}{8} \right]$$

$$= \frac{5\pi}{4} \text{ Satuan Luas}$$